Basics of AI and Machine Learning

State-Space Search: State Spaces

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State-Space Search Problems

Classical State-Space Search Problems Informally

(Classical) state-space search problems are among the "simplest" and most important classes of AI problems.

objective of the agent:

- from a given initial state
- apply a sequence of actions
- in order to reach a goal state

performance measure: minimize total action cost

Motivating Example: 15-Puzzle

9	2	12	6
5	7	14	13
3		1	11
15	4	10	8

→

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Classical Assumptions

"classical" assumptions:

- no other agents in the environment (single-agent)
- always knows state of the world (fully observable)
- state only changed by the agent (static)
- finite number of states/actions (in particular discrete)
- actions have deterministic effect on the state

$\rightsquigarrow$ can all be generalized

For simplicity, we omit "classical" in the following.

Classification

Classification:

State-Space Search

environment:

- static vs. dynamic
- deterministic vs. non-deterministic vs. stochastic
- fully vs. partially vs. not observable
- discrete vs. continuous
- single-agent vs. multi-agent

problem solving method:

- problem-specific vs. general vs. learning

Search Problem Examples

- toy problems: combinatorial puzzles (Rubik's Cube, 15-puzzle, towers of Hanoi, ...)
- scheduling of events, flights, manufacturing tasks
- query optimization in databases
- behavior of NPCs in computer games
- code optimization in compilers
- verification of soft- and hardware
- sequence alignment in bioinformatics
- route planning (e.g., Google Maps)
- ...

thousands of practical examples

State-Space Search: Overview

Chapter overview: state-space search

- Foundations
  - State Spaces
  - Representation of State Spaces
  - Examples of State Spaces
- Basic Algorithms
- Heuristic Algorithms

Formalization

Formalization

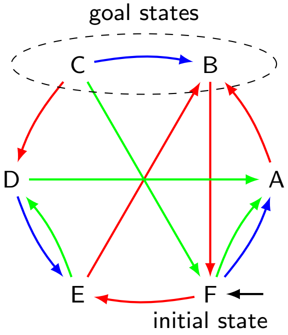
preliminary remarks:

- to cleanly study search problems we need a formal model
- fundamental concept: state spaces
- state spaces are (labeled, directed) graphs
- paths to goal states represent solutions
- shortest paths correspond to optimal solutions

State Spaces: Example

State spaces are often depicted as directed graphs.

- states: graph vertices
- transitions: labeled arcs
  (here: colors instead of labels)
- initial state: incoming arrow
- goal states: marked
  (here: by the dashed ellipse)
- actions: the arc labels
- action costs: described separately
  (or implicitly = 1)



State Spaces

Definition (state space)

A state space or transition system is a 6-tuple $\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$ with

- S: finite set of states
- A: finite set of actions
- $cost : A \to \mathbb{R}_0^+$ action costs
- $T \subseteq S \times A \times S$ transition relation; deterministic in $\langle s, a \rangle$ (see next slide)
- $s_0 \in S$ initial state
- $S_\star \subseteq S$ set of goal states

State Spaces: Transitions, Determinism

Definition (transition, deterministic)

Let $\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$ be a state space.

The triples $\langle s, a, s' \rangle \in T$ are called (state) transitions.

We say $\mathcal{S}$ has the transition $\langle s, a, s' \rangle$ if $\langle s, a, s' \rangle \in T$.
We write this as $s \xrightarrow{a} s'$, or $s \to s'$ when a does not matter.

Transitions are deterministic in $\langle s, a \rangle$: it is forbidden to have both $s \xrightarrow{a} s_1$ and $s \xrightarrow{a} s_2$ with $s_1 \neq s_2$.

State Spaces: Terminology

We use common terminology from graph theory.

Definition (predecessor, successor, applicable action)

Let $\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$ be a state space.

Let $s, s' \in S$ be states with $s \to s'$.

- s is a predecessor of s'
- s' is a successor of s

If $s \xrightarrow{a} s'$, then action a is applicable in s.

State Spaces: Terminology

We use common terminology from graph theory.

Definition (path)

Let $\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$ be a state space.

Let $s^{(0)}, \dots, s^{(n)} \in S$ be states and $\pi_1, \dots, \pi_n \in A$ be actions such that $s^{(0)} \xrightarrow{\pi_1} s^{(1)}, \dots, s^{(n-1)} \xrightarrow{\pi_n} s^{(n)}$.

- $\pi = \langle \pi_1, \dots, \pi_n \rangle$ is a path from $s^{(0)}$ to $s^{(n)}$
- length of π: $|\pi| = n$
- cost of π: $cost(\pi) = \sum_{i=1}^{n} cost(\pi_i)$

- paths may have length 0

State Spaces: Terminology

more terminology:

Definition (reachable, solution, optimal)

Let $\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$ be a state space.

- state s is reachable if a path from s_0 to s exists
- paths from $s \in S$ to some state $s_\star \in S_\star$ are solutions for/from s
- solutions for s_0 are called solutions for $\mathcal{S}$
- optimal solutions (for s) have minimal costs among all solutions (for s)

State-Space Search

State-Space Search

State-Space Search

State-space search is the algorithmic problem of finding solutions in state spaces or proving that no solution exists.

In optimal state-space search, only optimal solutions may be returned.

Summary

Summary

- classical state-space search problems:
  find action sequence from initial state to a goal state
- performance measure: sum of action costs
- formalization via state spaces:
  - states, actions, action costs, transitions,
    initial state, goal states
- terminology for transitions, paths, solutions
- definition of (optimal) state-space search