Basics of AI and Machine Learning

State-Space Search: Best-first Graph Search

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Best-first Search

Best-first Search

Best-first search is a class of search algorithms that expand the "most promising" node in each iteration.

- decision which node is most promising uses heuristics. . .
- . . . but not necessarily exclusively.

- implementation essentially like uniform cost search
- different choices of f $\rightsquigarrow$ different search algorithms

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Best-first Search

A best-first search is a heuristic search algorithm that evaluates search nodes with an evaluation function f and always expands a node n with minimal $f(n)$ value.

- implementation essentially like uniform cost search
- different choices of f $\rightsquigarrow$ different search algorithms

The Most Important Best-first Search Algorithms

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What do we obtain with $f(n) := g(n)$? $\rightsquigarrow$ uniform cost search

Best-first Search: Graph Search or Tree Search?

Best-first search can be graph search or tree search.

- here: graph search (i.e., with duplicate elimination), which is the more common case

Best-first Search

Best-first Search

```
$open$ := new MinHeap ordered by $\langle f, h \rangle$
if $h(\text{init}()) < \infty$:
    $open$.insert(make_root_node())
$closed$ := new HashSet
while not $open$.is_empty():
    $n$ := $open$.pop_min()
    if $n$.state $\notin$ $closed$:
        $closed$.insert($n$.state)
        if is_goal($n$.state):
            return extract_path($n$)
        for each $\langle a, s' \rangle \in$ succ($n$.state):
            if $h(s') < \infty$:
                $n'$ := make_node($n, a, s'$)
                $open$.insert($n'$)
return unsolvable
```

Best-first Search: Properties

properties:

- complete if h is safe: duplicate detection
- optimality depends on f

Greedy Best-first Search

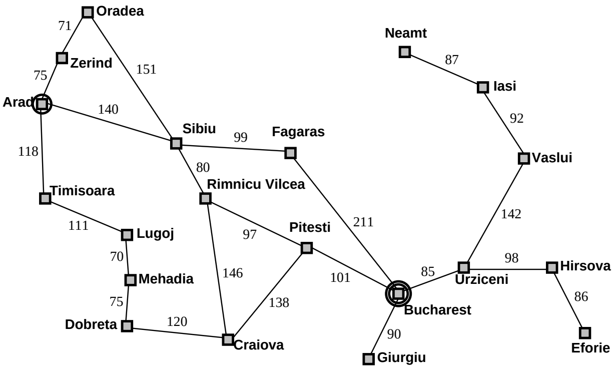
Greedy Best-first Search

Greedy Best-first Search

only consider the heuristic: $f(n) = h(n.\text{state})$

Note: usually without reopening (for reasons of efficiency)

Example: Greedy Best-first Search for Route Planning



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Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
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Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
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Sibiu	253
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(a) The initial state

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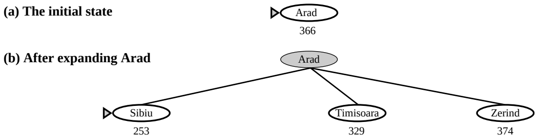

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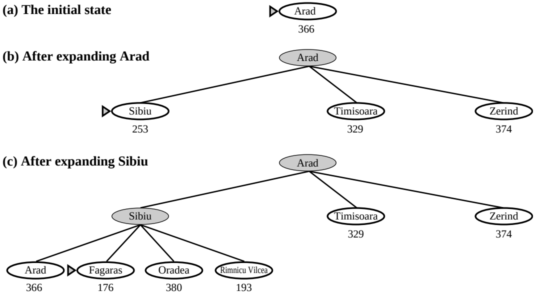

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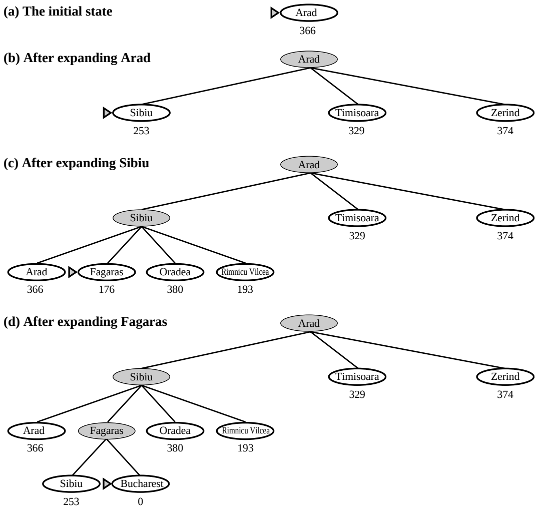


Greedy Best-first Search: Properties

- complete with safe heuristics
  (like all variants of best-first graph search)
- suboptimal: solutions can be arbitrarily bad
- often very fast: one of the fastest search algorithms in practice

A^*

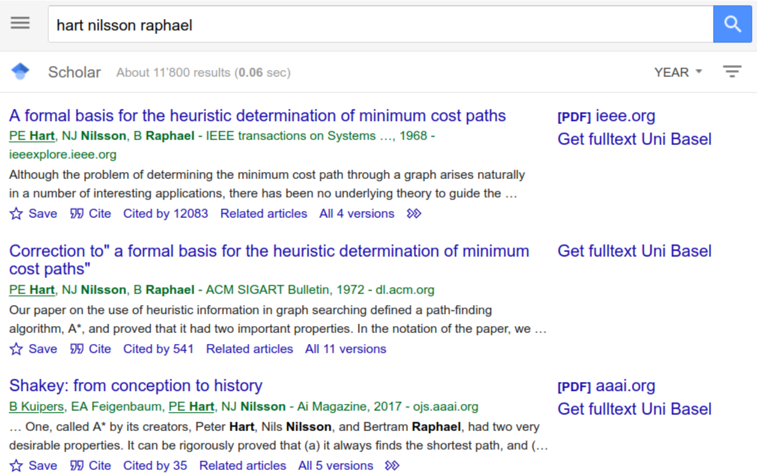
A*

A*

combine greedy best-first search with uniform cost search:
$f(n) = g(n) + h(n.\text{state})$

- trade-off between path cost and proximity to goal
- $f(n)$ estimates overall cost of cheapest solution from initial state via n to the goal

A*: Citations



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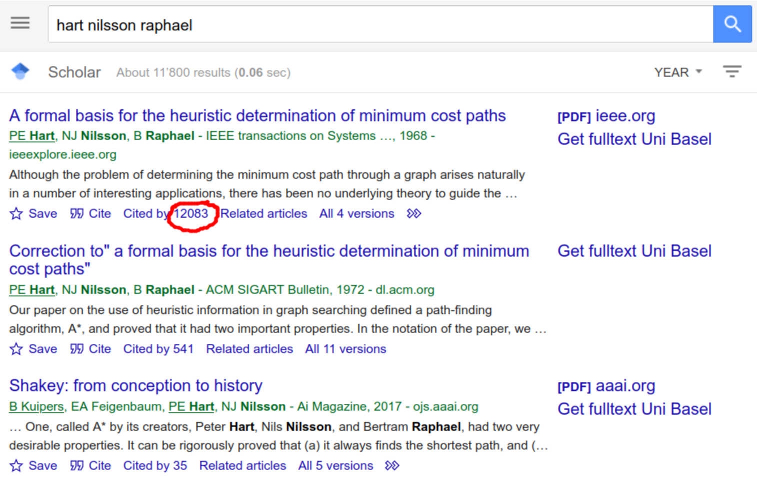

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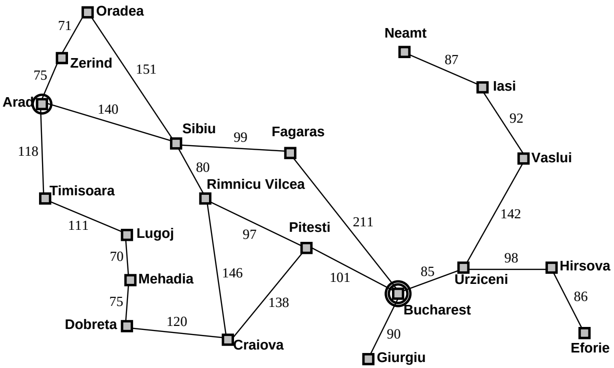

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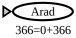

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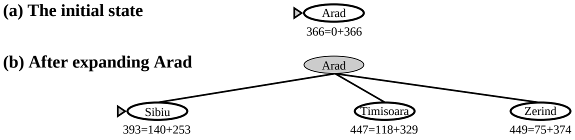

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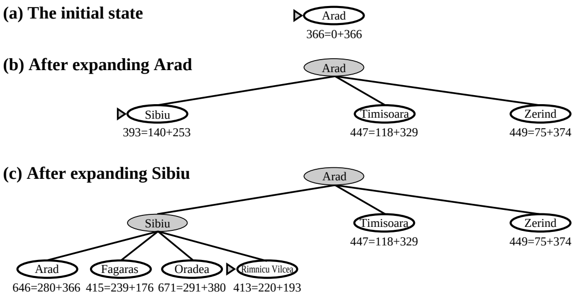

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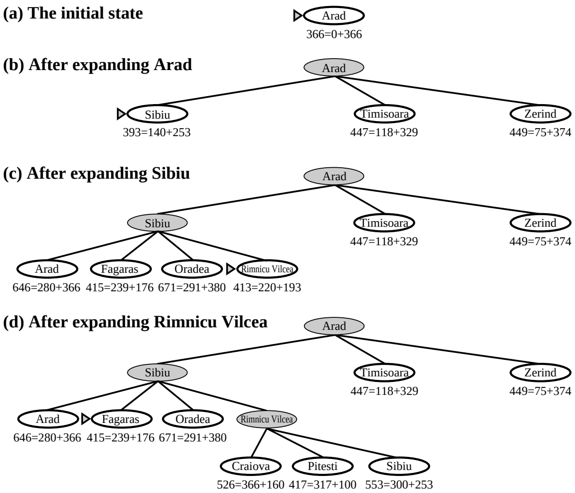

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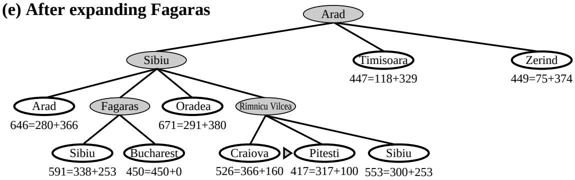

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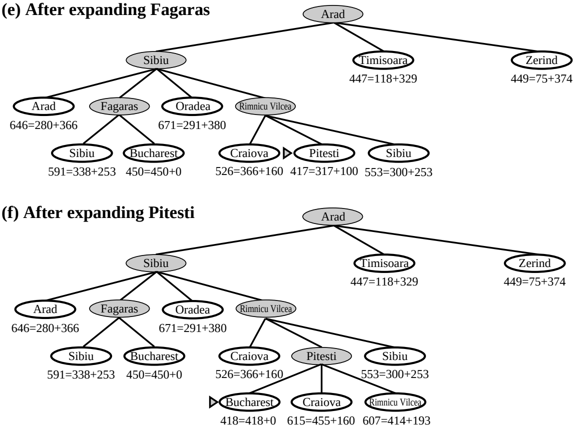

A*: Properties

- complete with safe heuristics
  (like all variants of best-first graph search)
- with reopening: optimal with admissible heuristics
- without reopening: optimal with heuristics
  that are admissible and consistent

Weighted A*

Weighted A*

Weighted A*

A* with more heavily weighted heuristic:

$f(n) = g(n) + w \cdot h(n.\text{state})$,

where weight $w \in \mathbb{R}_0^+$ with $w \geq 1$ is a freely choosable parameter

Weighted A^*: Properties

weight parameter controls "greediness" of search:

- $w = 0$: like uniform cost search
- $w = 1$: like A^*
- $w \to \infty$: like greedy best-first search

with $w \geq 1$ properties analogous to A^*:

- *h* admissible:
  found solution guaranteed to be at most w times as expensive as optimum when reopening is used

Summary

Summary

best-first graph search with evaluation function f:

- $f = h$: greedy best-first search
  suboptimal, often very fast
- $f = g + h$: A^*
  optimal if h admissible and consistent
- $f = g + w \cdot h$: weighted A^*
  for $w \geq 1$ suboptimality factor at most w
  under same conditions as for optimality of A^*