

# TDDE56: Problem Solving as Search

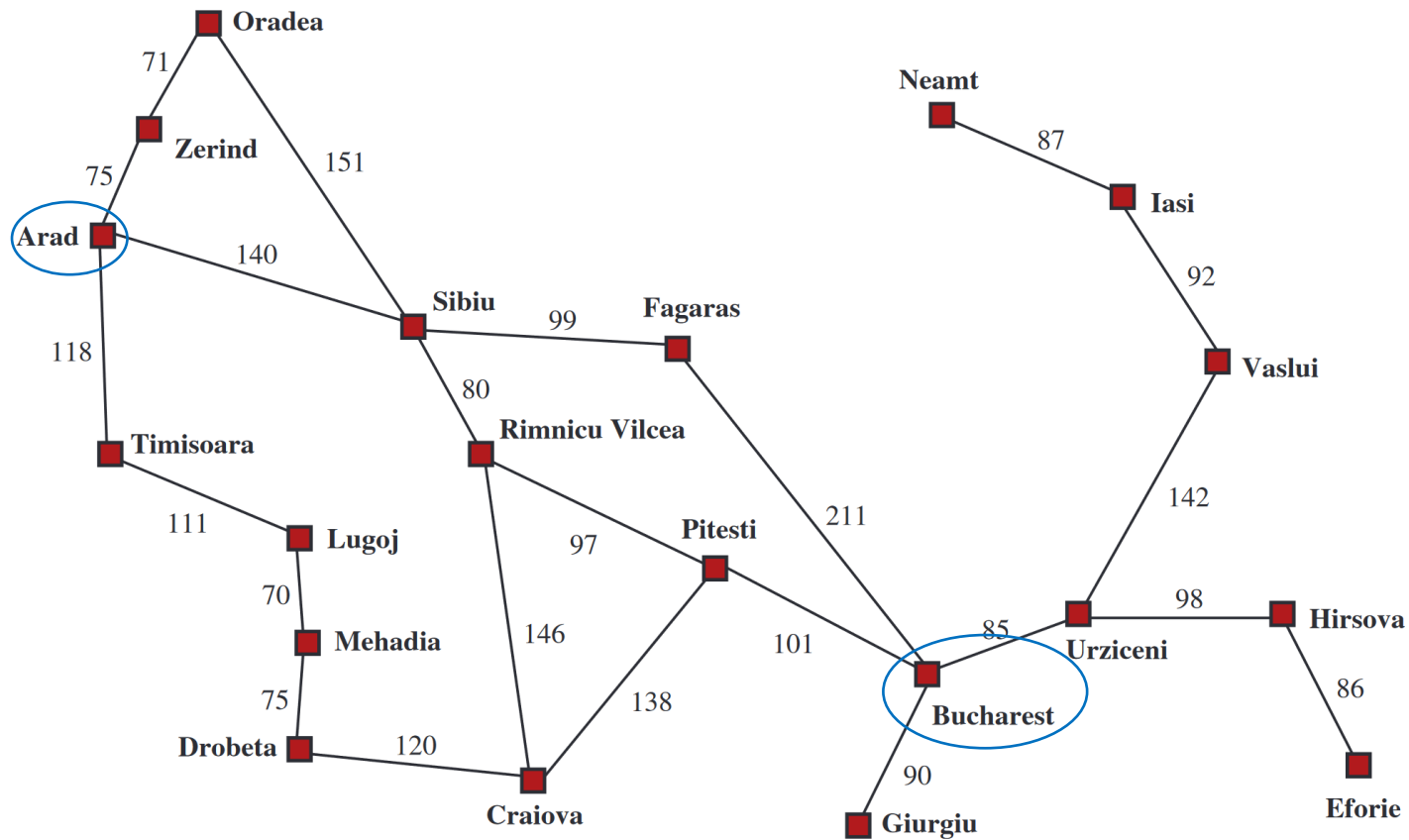
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@FredrikHeintz

# Problem-Solving through Search



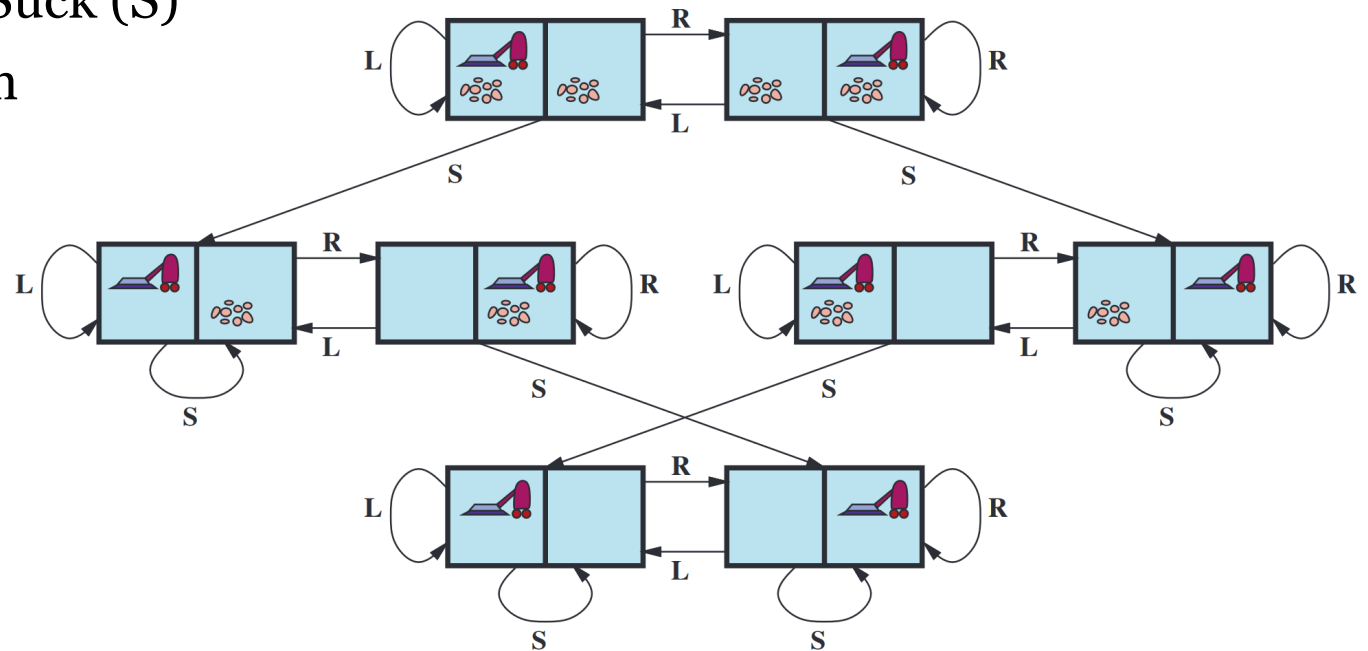
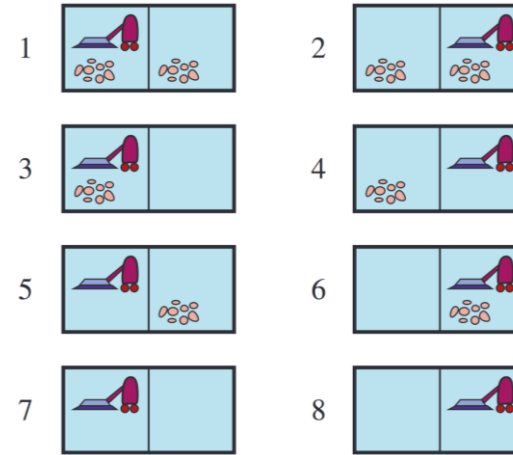
**Figure 3.1** A simplified road map of part of Romania, with road distances in miles.

# Search – Problem Definition

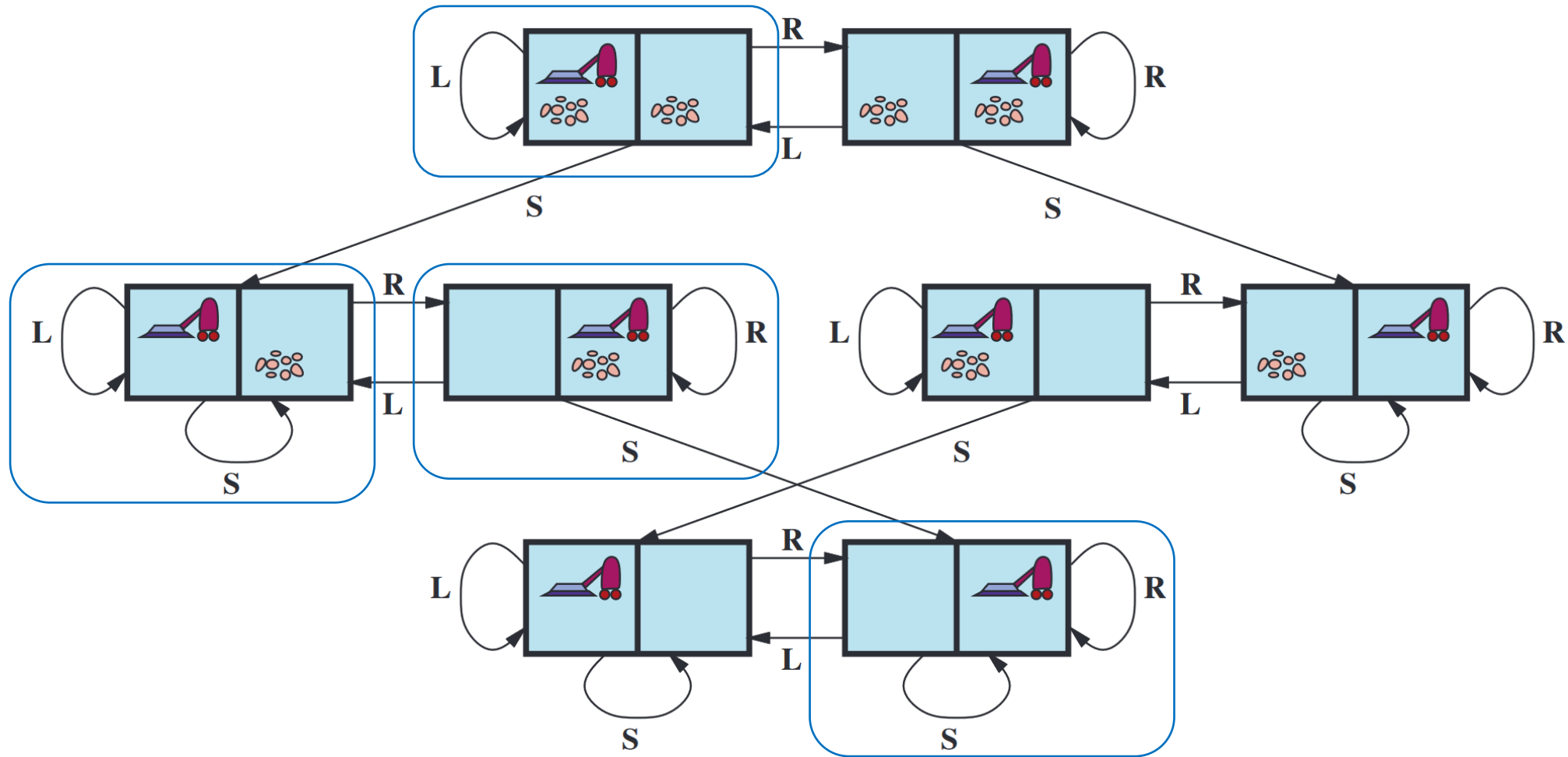
- **Initial State:** The state in which the agent starts or initial condition of the agent.
- **States:** All states that are reachable from initial state by any sequence of actions or all possible states that the agent can take. This is also referred to as State space.
- **Actions:** All possible actions that the agent can execute. Specifically, it provides the list of actions, that an agent can perform in a particular state. This is also referred to as Action space.
- **Transition Model:** This property describes the results of each action taken in a particular state.
- **Goal Test:** A way to check, whether a state is the goal.
- **Path Cost:** A function that assigns a numeric cost to a path w.r.t. performance measure

# Vacuum World

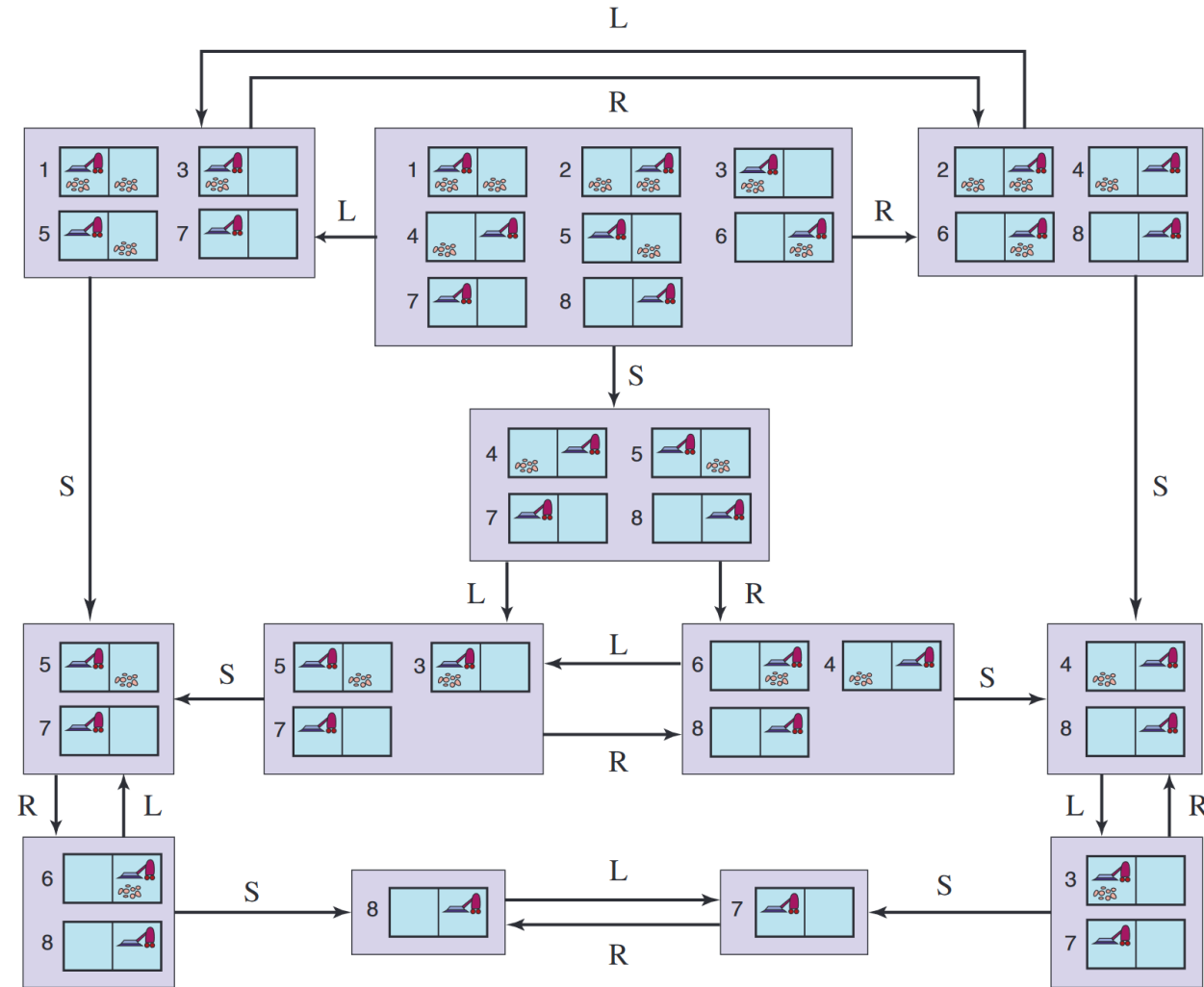
- State Space: 2 positions, dirt or no dirt
- Initial State: Choose
- Goal States: States with no dirt in the rooms
- Actions: Left (L), Right (R), or Suck (S)
- Action costs: one unit per action
- Transition model:



# Solving the Vacuum World

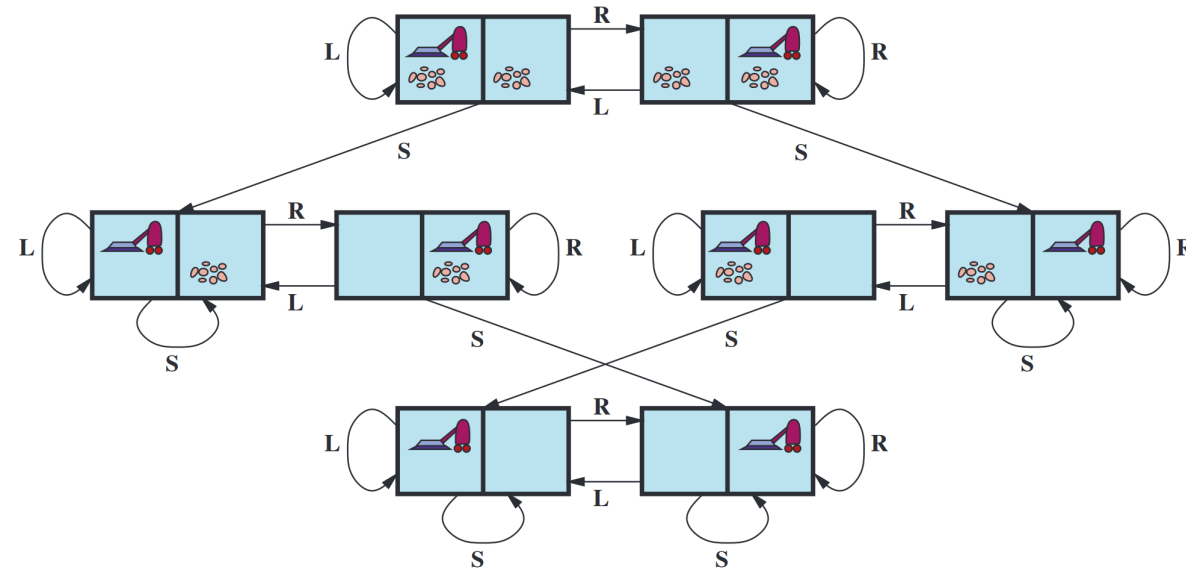
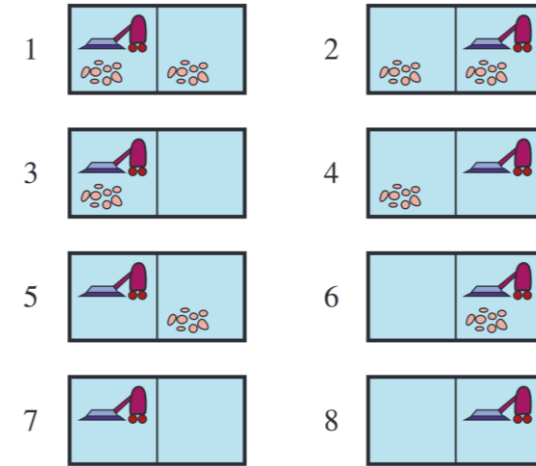


# Solving the Vacuum World without Sensors



# Search – Search Space

- **State space:** physical configuration
- **Search space:** abstract configuration often represented by a search tree or graph where a path is a possible solution.
- **Search tree:** representation of configurations and how they are connected by actions. A path represents a sequence of actions. The *root* is the initial state. The actions taken make the *branches* and the *nodes* are results of those actions. A node has depth, path cost and associated state in the state space.



# Search Tree Construction

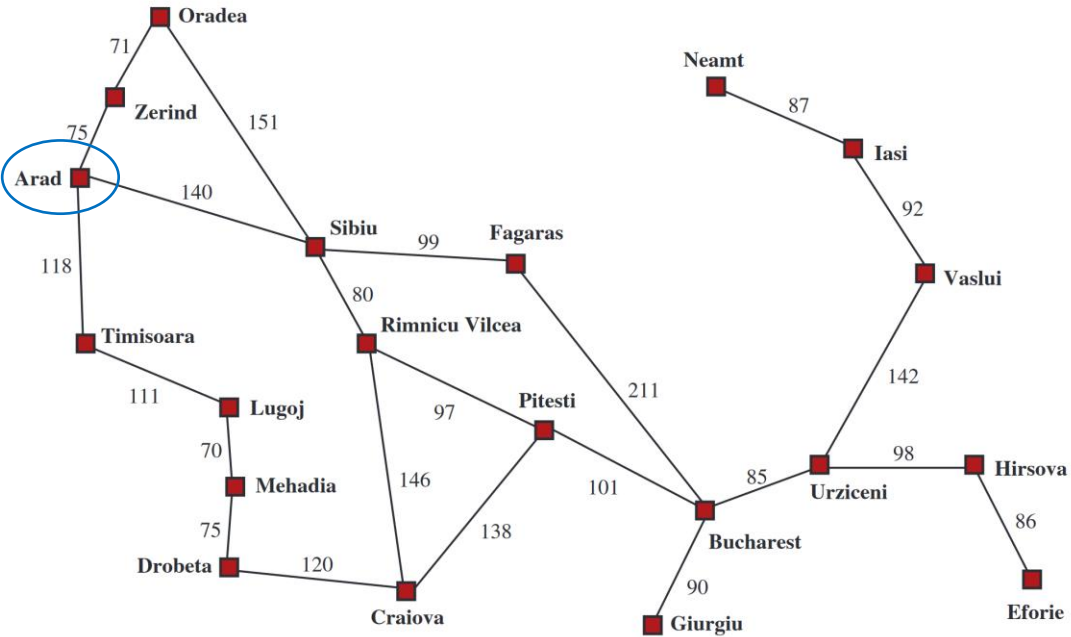
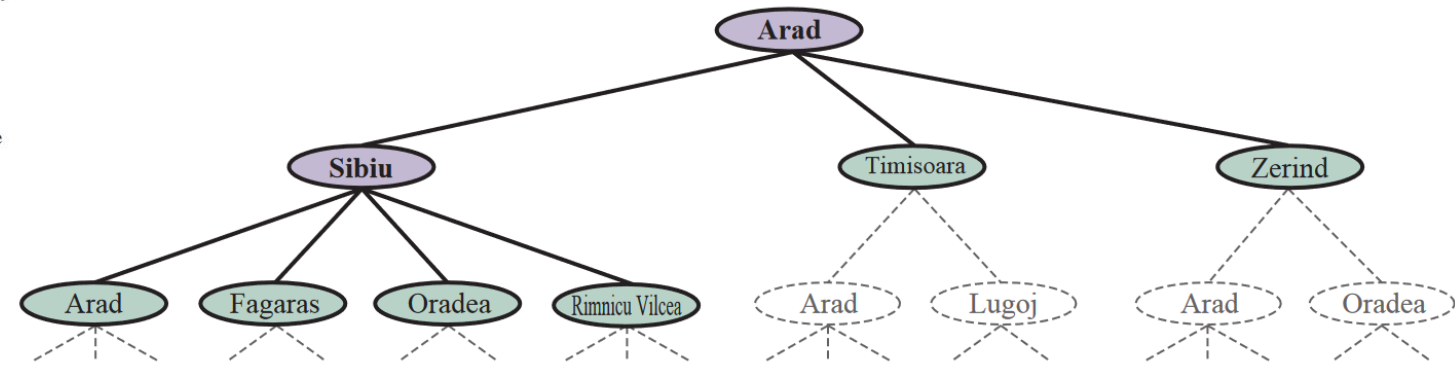
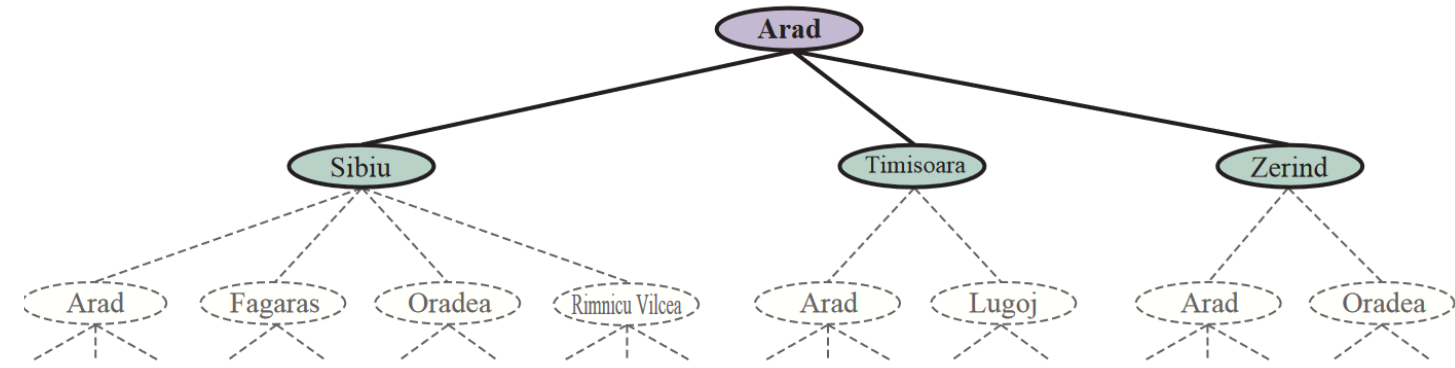
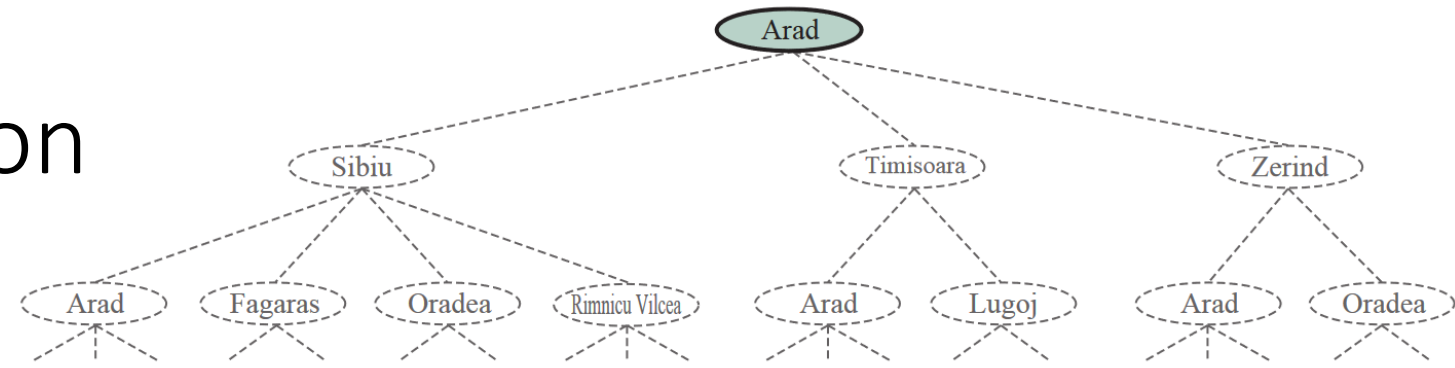


Figure 3.1 A simplified road map of part of Romania, with road distances in miles.



# Search Strategies

- A *strategy* is defined by picking the order of node expansion.
- Strategies can be *evaluated* along the following dimensions:
  - *Completeness* – does it find a solution if it exists?
  - *Time Complexity* – number of nodes generated/expanded
  - *Space Complexity* – maximum number of nodes in memory
  - *Optimality* – does it always find a least cost solution
- Time and space complexity are *measured* in terms of:
  - $b$  – *maximum branching* factor of search tree
  - $d$  – *depth of the least cost solution* in the search tree
  - $m$  – *maximum length* of any path in the state space (possibly infinite)

# Some Search Classes

- *Uninformed Search* (Blind Search)
  - No additional information about states besides that in the problem definition
  - Can only generate successors and compare against state.
  - Some examples:
    - Breadth-first search, Depth-first search, Iterative deepening DFS
- *Informed Search* (Heuristic Search)
  - Strategies have additional information as to whether non-goal states are more promising than others.
  - Some examples:
    - Greedy Best-First Search, A\* Search

# Breadth-First Search

**function** BREADTH-FIRST-SEARCH(*problem*) **returns** a solution node or *failure*

*node*  $\leftarrow$  NODE(*problem*.INITIAL)

**if** *problem*.IS-GOAL(*node*.STATE) **then return** *node*

*frontier*  $\leftarrow$  a FIFO queue, with *node* as an element

*reached*  $\leftarrow$  {*problem*.INITIAL}

**while not** IS-EMPTY(*frontier*) **do**

*node*  $\leftarrow$  POP(*frontier*)

**for each** *child* **in** EXPAND(*problem*, *node*) **do**

*s*  $\leftarrow$  *child*.STATE

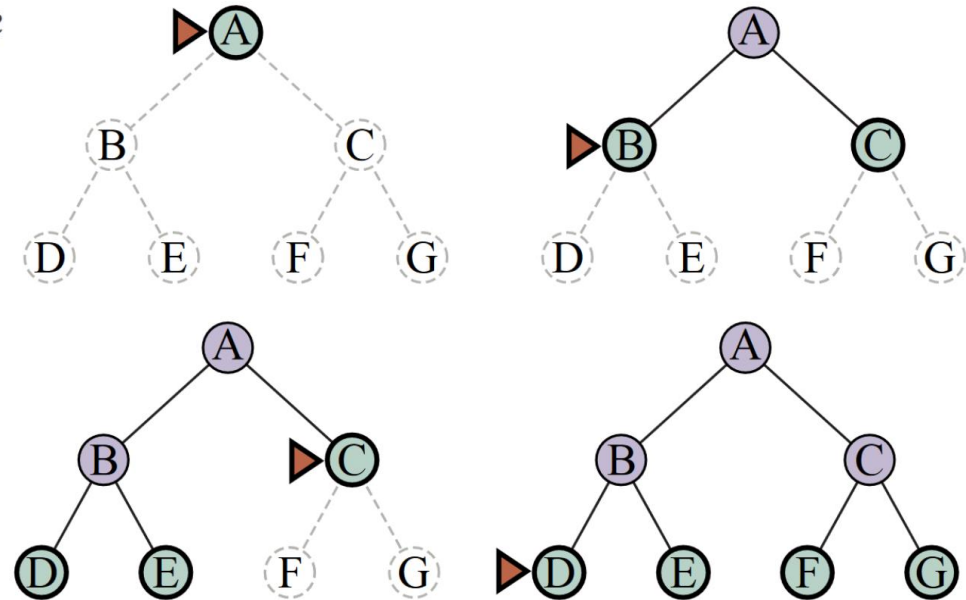
**if** *problem*.IS-GOAL(*s*) **then return** *child*

**if** *s* is not in *reached* **then**

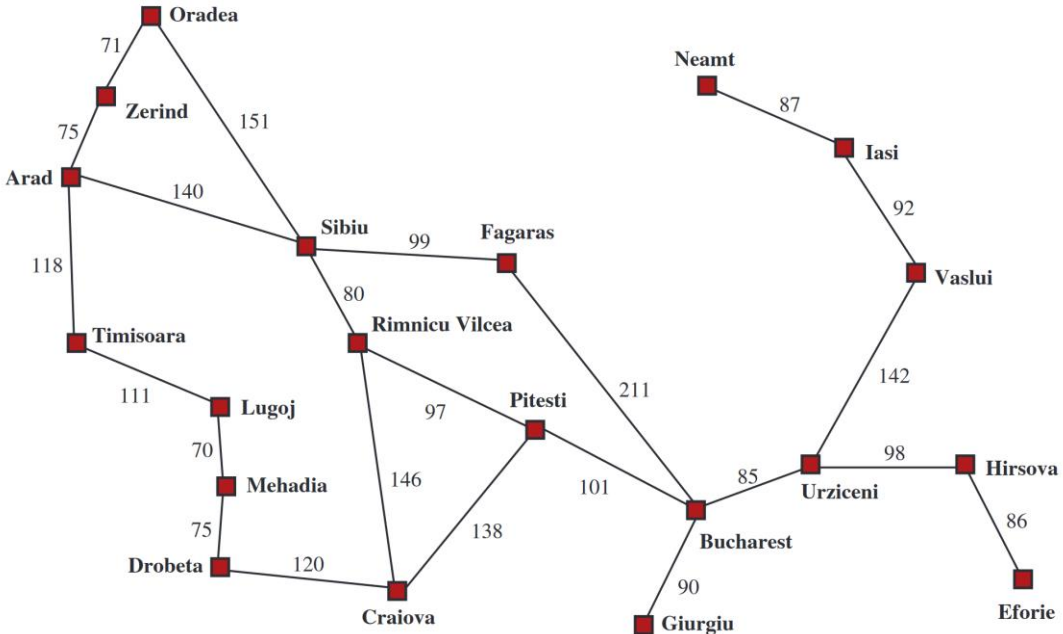
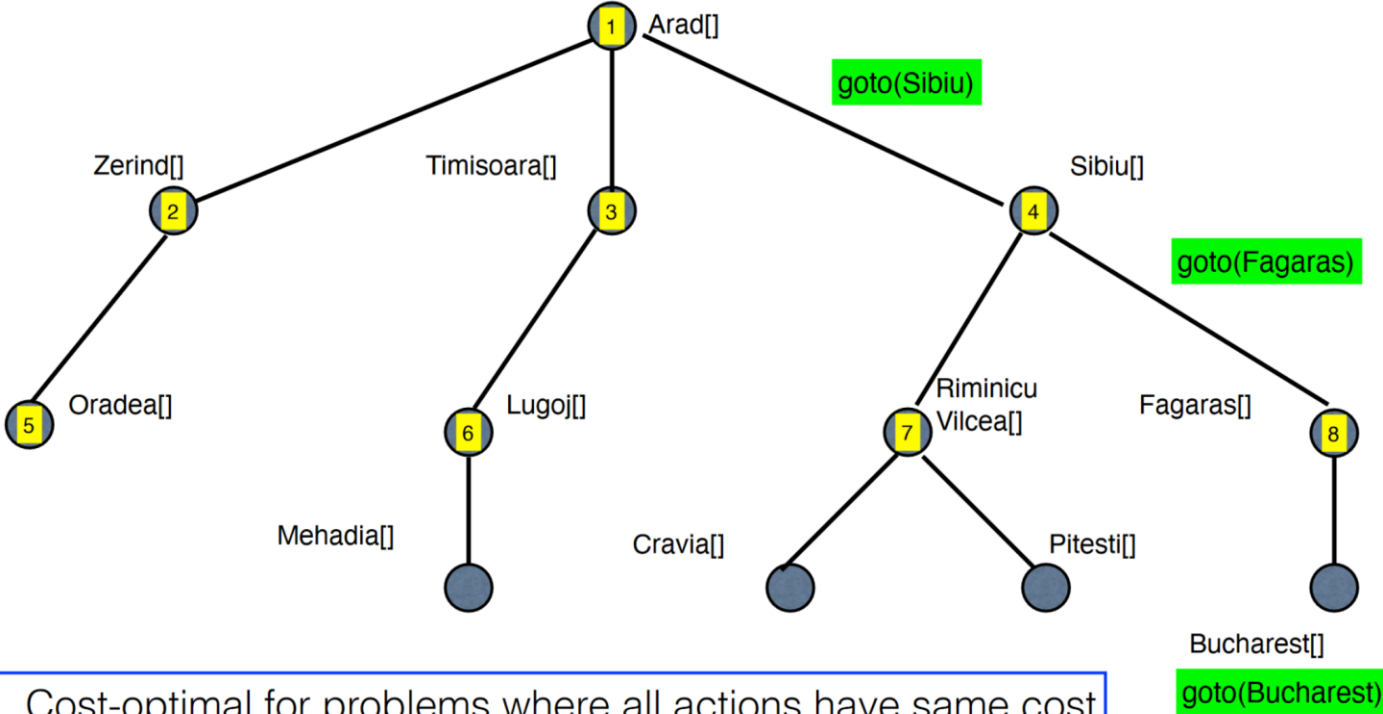
add *s* to *reached*

add *child* to *frontier*

**return** *failure*

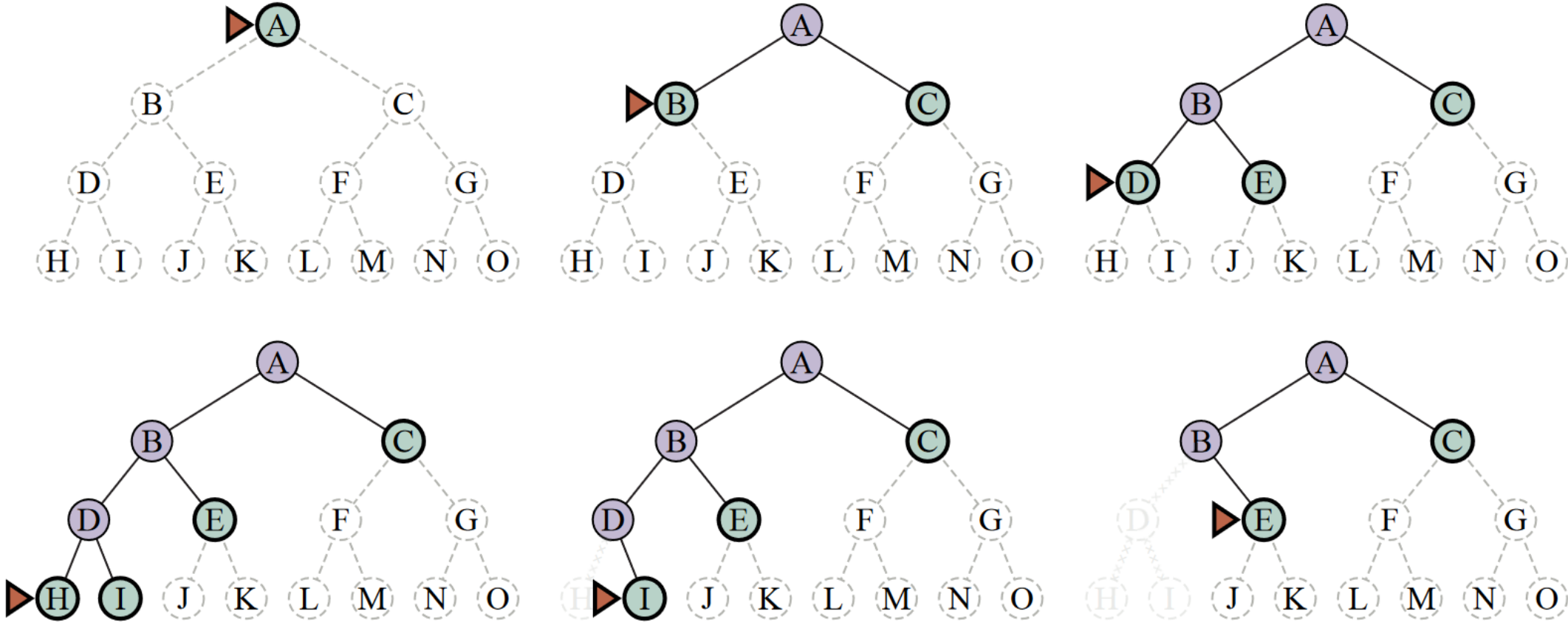


# Breadth-First Search

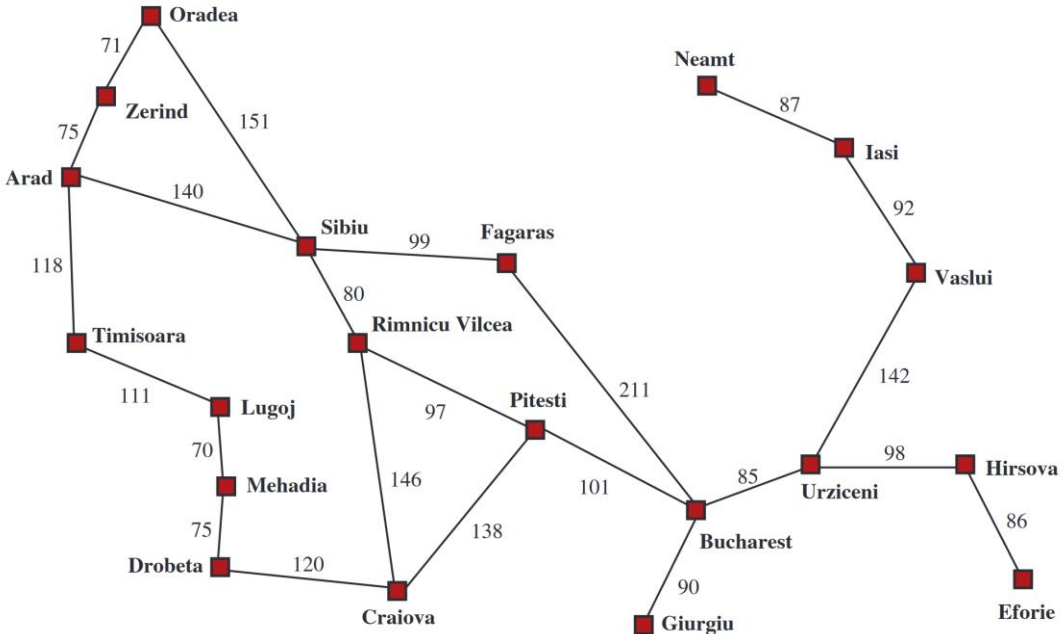
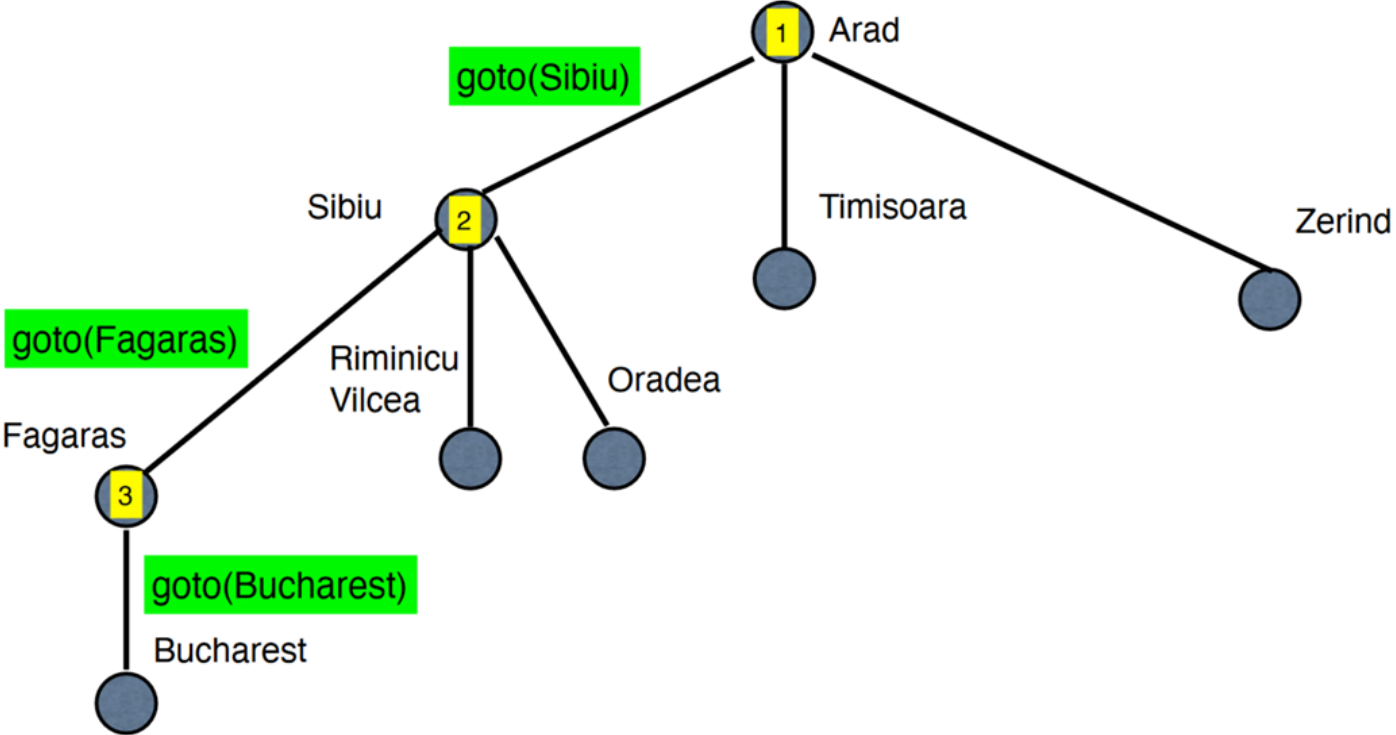


- Cost-optimal for problems where all actions have same cost
- Finds solution with minimal number of actions
- It is complete

# Depth-First Search



# Depth-First Search



# Heuristic Search

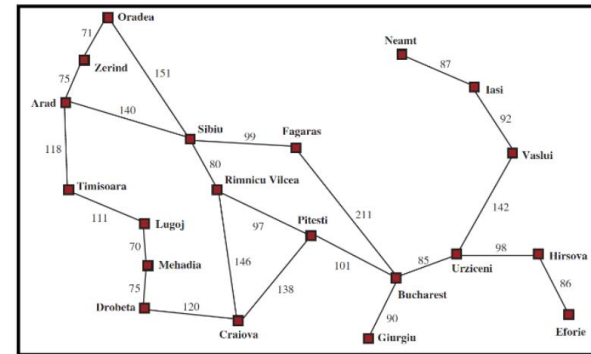
Straight line distance from city  $n$  to goal city  $n'$

Assume the cost to get somewhere is a function of the distance traveled

Straight line distance to Bucharest from any city

$h_{SLD}()$

<b>Arad</b>	366	<b>Mehadia</b>	241
<b>Bucharest</b>	0	<b>Neamt</b>	234
<b>Craiova</b>	160	<b>Oradea</b>	380
<b>Drobeta</b>	242	<b>Pitesti</b>	100
<b>Eforie</b>	161	<b>Rimnicu Vilcea</b>	193
<b>Fagaras</b>	176	<b>Sibiu</b>	253
<b>Giurgiu</b>	77	<b>Timisoara</b>	329
<b>Hirsova</b>	151	<b>Urziceni</b>	80
<b>Iasi</b>	226	<b>Vaslui</b>	199
<b>Lugoj</b>	244	<b>Zerind</b>	374



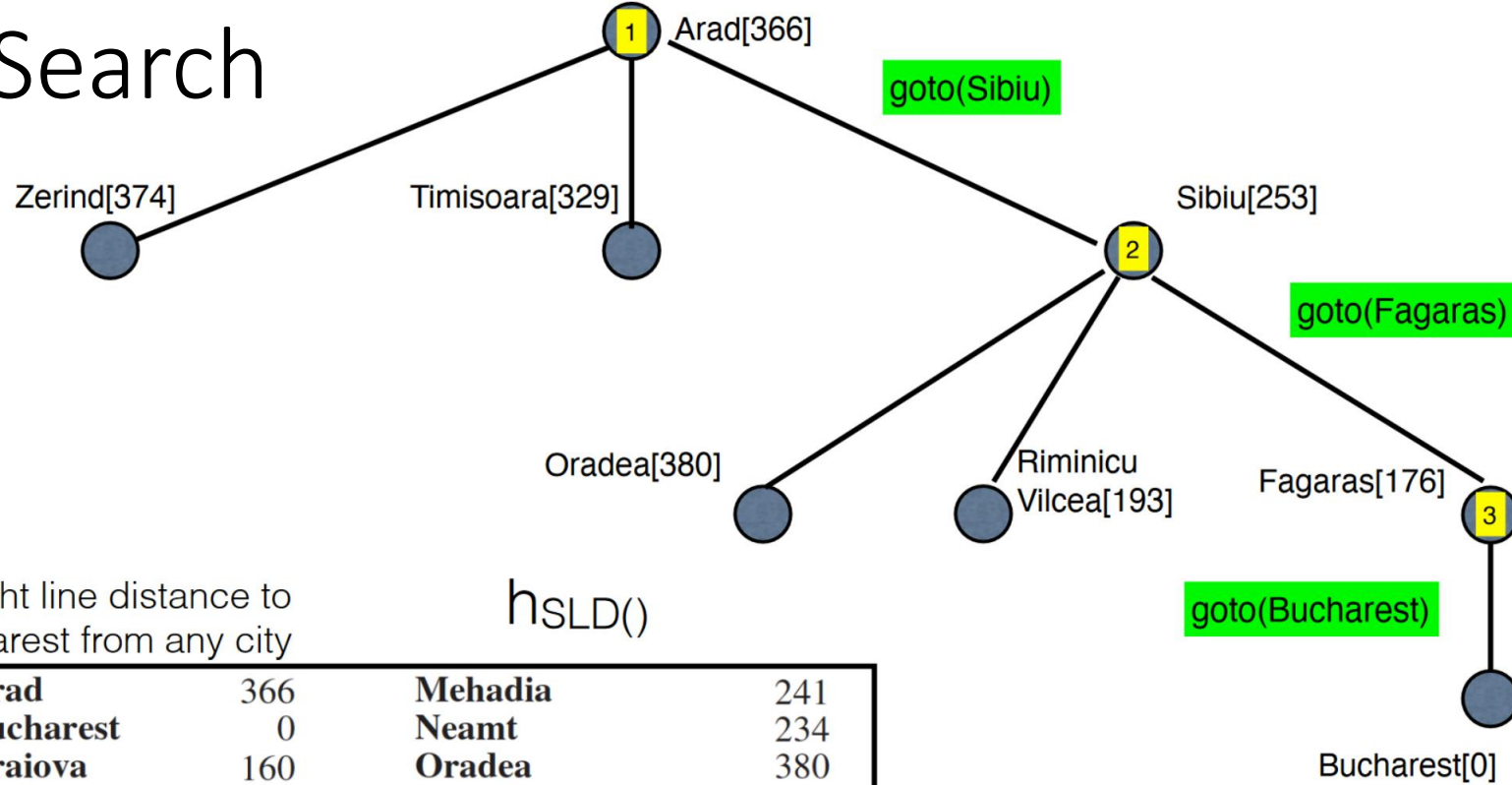
Heuristic:

$$f(n) = h_{SLD}(n)$$

Notice the SLD under estimates the actual cost!



# Heuristic Search



Straight line distance to Bucharest from any city

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# Adversarial Search – Minimax

**function** MINIMAX-SEARCH(*game, state*) **returns** an action

```

player ← game.TO-MOVE(state)
value, move ← MAX-VALUE(game, state)
return move
    
```

**function** MAX-VALUE(*game, state*) **returns** a (utility, move) pair

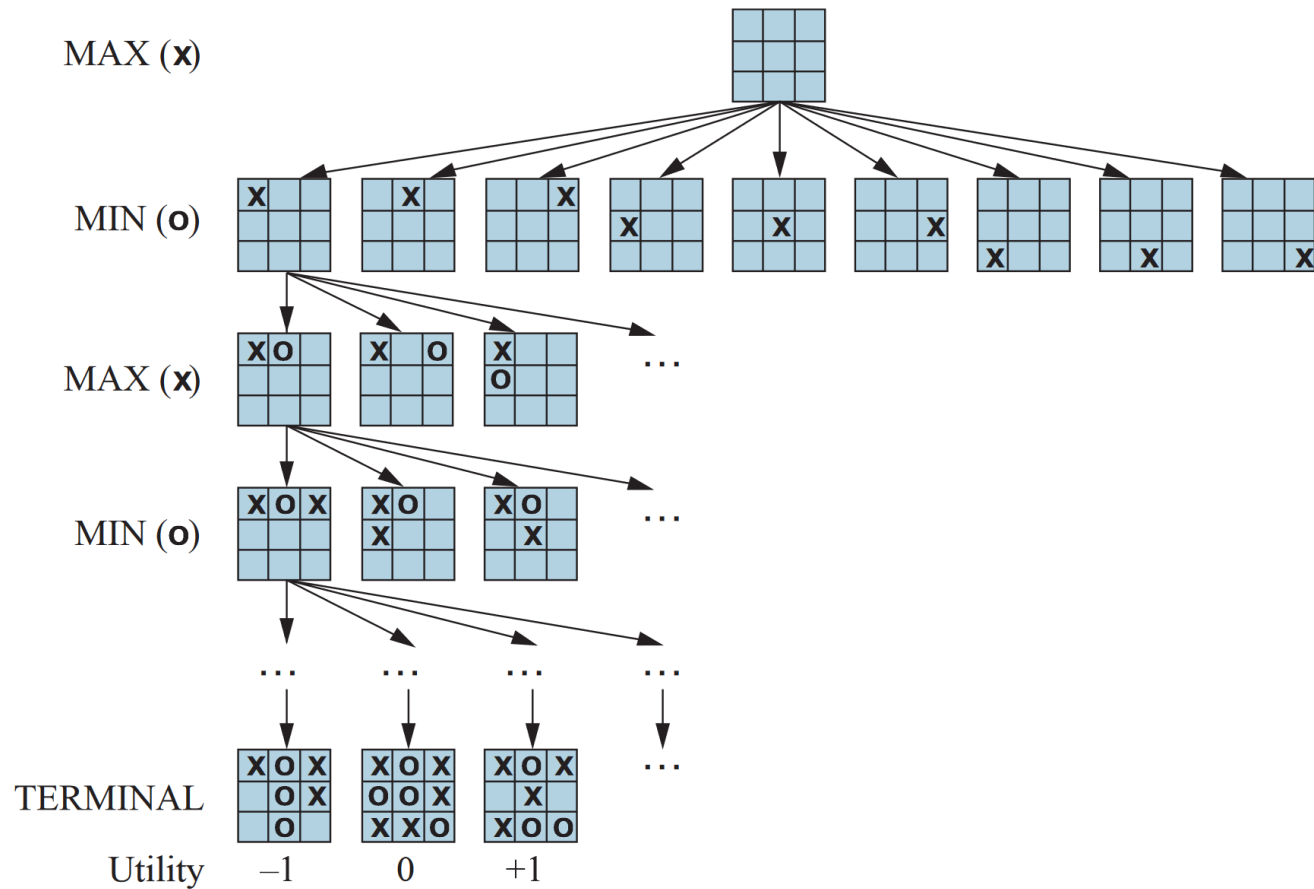
```

if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null
v ← -∞
for each a in game.ACTIONS(state) do
    v2, a2 ← MIN-VALUE(game, game.RESULT(state, a))
    if v2 > v then
        v, move ← v2, a
return v, move
    
```

**function** MIN-VALUE(*game, state*) **returns** a (utility, move) pair

```

if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null
v ← +∞
for each a in game.ACTIONS(state) do
    v2, a2 ← MAX-VALUE(game, game.RESULT(state, a))
    if v2 < v then
        v, move ← v2, a
return v, move
    
```



# Applications of Search

- Game playing (chess, Go, ...)
- Constraint satisfaction
- Optimization
- Machine learning
- Planning
- ...